Logistics Management Institute

An Upper Bound for Optimal Maintenance Costs of Weapon Systems

A Technical Note

IR527LN1

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An Upper Bound for Optimal Maintenance Costs of Weapon Systems

1.0 Introduction

New weapon systems being designed and fielded are composed of sophisticated and highly integrated subsystems that use computers, printed circuit boards, and complex electronic and electro-optical equipment. Typically they are modular and utilize a remove-and-replace maintenance concept. If the components are relatively inexpensive, we can buy enough spare components and subcomponents for the proper maintenance of weapon systems. However, since high-technology components are usually more expensive than mechanical ones, the maintenance costs of complex weapon systems can be much higher than those of simpler weapon systems. One of the most important tasks of logisticians is to determine the level of optimal maintenance costs so that maintenance resources can be used efficiently.

In this report we propose a way to determine an upper bound for the optimal maintenance costs of a weapon system and for the optimal allocation of maintenance resources and identification of problems in maintaining weapon systems, while having available the target number of weapon systems in the field. The maintenance costs in this study include the costs of repair parts and labor costs for maintenance actions.

2.0 Model for the Optimal Maintenance Cost

2.1 Notations

P: weapon system procurement cost per unit

T: lifetime of the weapon system

x: lifetime maintenance cost ratio to the price of the system P (total lifetime maintenance cost per end item is Px)

$A(x)$: maximum average availability of the weapon system achieved with lifetime maintenance cost ratio x

n: target number of available weapon systems

$N(x)$: minimum number of weapon systems required in order to have at least n available weapon systems in the field

$C(x)$: total lifetime costs.

2.2 Model

- Costs
  - Investment cost: $N(x)P$
  - Lifetime maintenance cost: nPx
  - Total cost: $N(x)P + nPx$.
- Constraints
  - $A(x)\,N(x) = n$ = target number of available systems
- Objective function
  - Minimize $C(x) = \dfrac{np}{A(x)} + nPx.$

2.3 Properties of $A(x)$ and $C(x)$

Since we define $A(x)$ as the maximum average availability of the weapon system achieved with lifetime maintenance cost ratio x, $A(x)$ is a continuous function of x. We assume that $A(x)$ is differentiable or that $A(x)$ can be approximated by a differentiable function.

In many cases, especially for high-tech weapon systems, the availability function $A(x)$ is S-shaped, as depicted in Figure 1. But we can assume, as shown later, that the logarithm, ln $(A(x))$, is concave.

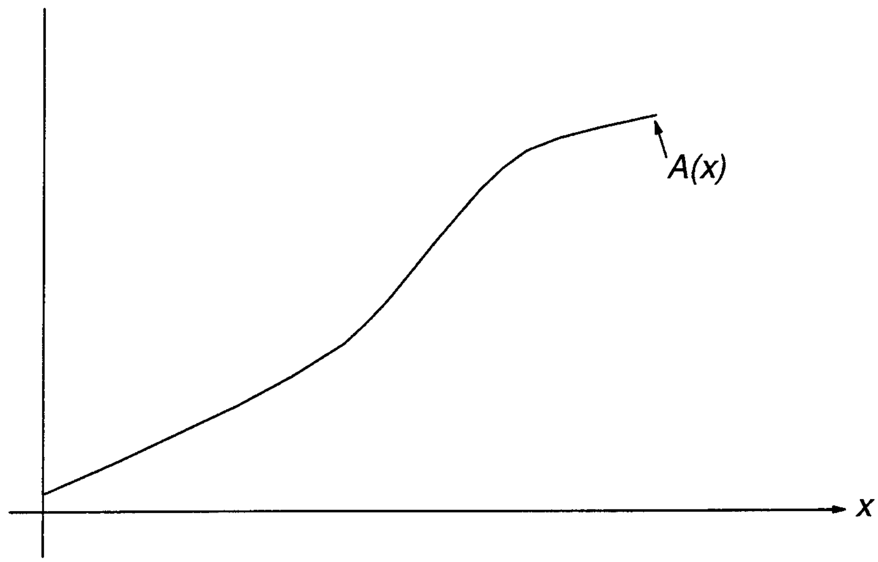

Figure 1.
Shape of Availability Function A(x)

Let $A_i(x_i)$ be the availability of component i of the system when we use x_i units of component i for each system. The component availability increases as we spend more maintenance resources, but the marginal effect of each additional unit decreases and is small compared to the current component availability, as shown in Figure 2.

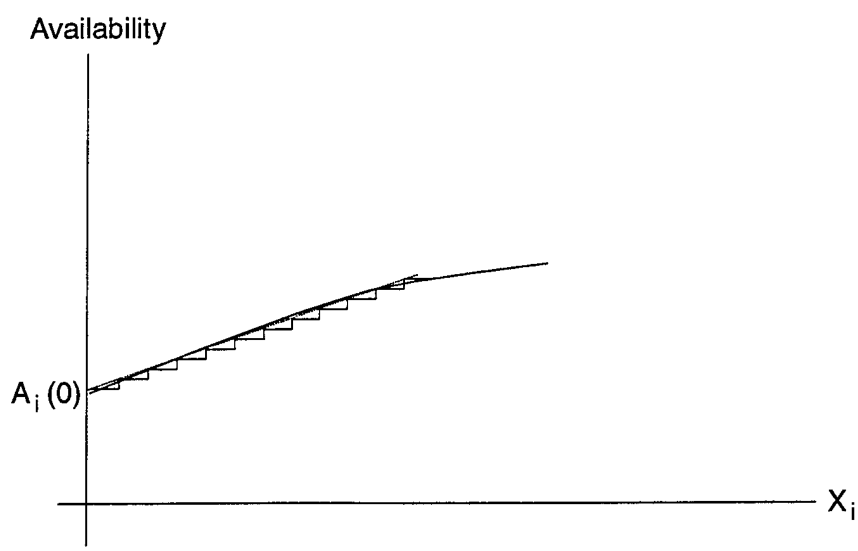

Figure 2.
Component Availability Function $A_i(x_i)$

Let X_i be the total number of component i allocated for the maintenance of weapon systems in the field. Then the component availability function A_i is a step function of X_i, or $x_i = \frac{X_i}{n}$. For $X_i = 0$, $A_i(0)$ is the average availability of the components in the system. If n is large, $A_i(0)$ is much higher than the marginal increase of the component availability for the additional one unit of component i. So, if n is large enough (say, greater than 100), then the component availability is approximately concave, as shown in Figure 2. Thus we can assume that $A_i(x_i)$ is a concave function of x_i.

Now, the system availability $A(x)$ is the product of component availability functions $A_i(x_i)$. That is

$$A(x) = \prod_i A_i(x_i).$$

Let P_x be the following optimization problem:

$$\begin{aligned} \ln A(x) &= \max \sum_i \ln A_i(x_i) \\ & \quad s.t. \sum_i c_i x_i \leq x, \end{aligned}$$

where c_i is the unit price of component i.

Let x_i^* and y_i^* be the optimal solutions of P_x and P_y respectively, and let $t_i = \alpha x_i^* + (1-\alpha) y_i^*$. Then

$$\begin{aligned} \sum_i c_i t_i &= \sum_i [\alpha x_i^* + (1-\alpha) y_i^*] \\ &= \alpha \sum_i x_i^* + (1-\alpha) \sum_i y_i^* \\ &\leq \alpha\, x + (1-\alpha) y, \end{aligned}$$

and hence (t_i) is a feasible solution of the optimization problem $P_{\alpha x + (1-\alpha) y}$. If $A_i(x_i)$ is concave, then $\ln A_i(x_i)$ is also concave and we have

$$\begin{aligned} \ln A(\alpha x + (1-\alpha) y) &\geq \sum_i \ln A_i(t_i) \\ &= \sum_i \ln A_i[\alpha x_i^* + \alpha(1-\alpha) y_i^*] \\ &\geq \sum_i [\alpha \ln A_i(x_i^*) + (1-\alpha) \ln A_i(y_i^*)] \\ &= \alpha \sum_i \ln A_i(x_i^*) + (1-\alpha) \sum_i \ln A_i(y_i^*) \\ &= \alpha \ln A(x) + (1-\alpha) \ln A(y) \end{aligned}$$

for any $x \geq 0$, $y \geq 0$, and $0 \leq \alpha \leq 1$. Therefore, $\ln A(x)$ is a concave function of x if all $A_i(x_i)$ are concave.

Now we can show that the cost function $C(x)$ is a convex function of x if $\ln A(x)$ is concave. Let $g(x) = \ln A(x)$. Then

$$g''(x) = \frac{A'' A(x) - [A'(x)]^2}{A^2(x)} \leq 0 \text{ for all } x \geq 0$$

or

$$A''(x) A(x) \leq [A'(x)]^2 \text{ for all } x \geq 0.$$

Since $C(x) = \frac{nP}{A(x)} + nPx$, we have

$$\begin{aligned} C''(x) &= \frac{-A''(x) A(x) + 2[A'(x)]^2}{A^3(x)} \bullet nP \\ &\geq \frac{[A'(x)]^2}{A^3(x)} nP \\ &\geq 0 \end{aligned}$$

for all $x \geq 0$. Thus, $C(x)$ is a convex function of x.

2.4 Optimal Maintenance Cost Ratio

The total cost $C(x)$ can be expressed by

$$\begin{aligned} C(x) &= N(x) P + nPx \\ &= nP + [N(x) - n] P + nPx \\ &= nP + \left[\frac{n}{A(x)} - nP\right] + nPx \qquad \text{[Eq. 1]} \\ &= nP + \frac{1 - A(x)}{A(x)} nP + nPx \\ &= \text{cost of } T/E + \text{cost of } M/F + \text{lifetime maintenance cost} \end{aligned}$$

where T/E is Table of Equipment and M/F is Maintenance Float. By differentiating Equation 1 with respect to x, we find that the optimal maintenance cost ratio x^* must satisfy.

$$\frac{A'(x^*)}{A^2(x^*)} = 1. \qquad \text{[Eq. 2]}$$

If we know the availability function $A(x)$, we can find the optimal maintenance cost ratio x^*. In most cases, however, the availability function $A(x)$ is unknown. We may have a few values of $A(x)$ for some values of x from the maintenance data of the systems in the field. But these values of $A(x)$ do not necessarily reflect the optimal uses of maintenance resources. So we cannot use this partial information in determining the optimal maintenance cost ratio.

3.0 An Upper Bound of the Optimal Maintenance Cost Ratio

3.1 Concave Availability Function $A(x)$

In this section, we find an upper bound of the optimal maintenance cost ratio when $A(x)$ is concave. Since $C(x)$ is convex, from the optimality condition of (Equation 2), we have the following:

$$\text{if } \frac{A'(x)}{A^2(x)} \leq 1, \text{ then } x^* \leq x. \qquad \text{[Eq. 3]}$$

So, if we could find a point x such that $\frac{A'(x)}{A^2(x)} < 1$, then an upper bound of x^* is x; that is, $x^* \leq x$.

Since $A(x)$ is concave, for any $x \geq 0$,

$$A'(x) \leq \frac{A(x)}{x} \text{ or } \frac{A'(x)}{A^2(x)} \leq \frac{1}{xA(x)} \qquad \text{[Eq. 4]}$$

as is shown in Figure 3.

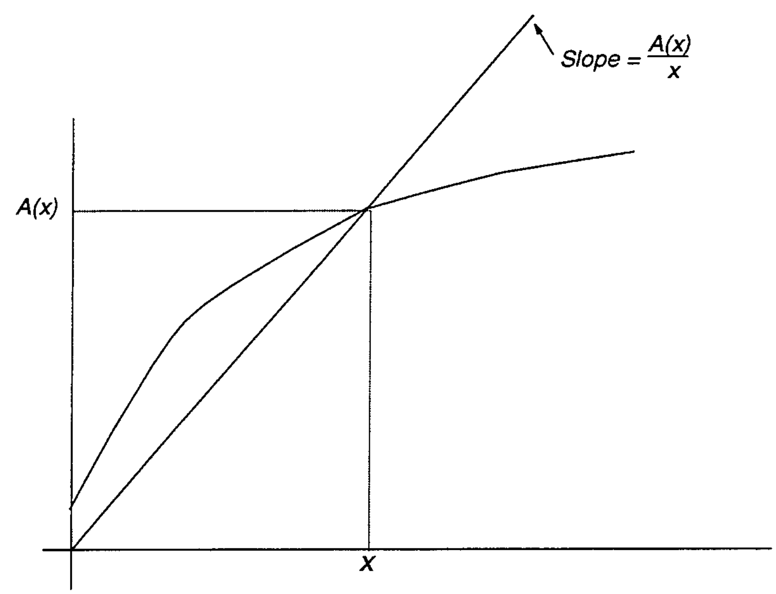

Figure 3.
Concave Availability Function

From Equation 3 and Equation 4, we have the following relationship:

$$\text{if } xA(x) \geq 1, \text{ then } x^* \leq x. \qquad \text{[Eq. 5]}$$

Let $(t, A(t))$ be a point such that $t\,A(t) > 1$. Then, for any x between 0 and t, as is shown in Figure 4,

$$A(x) \geq \frac{A(t)x}{t}.$$

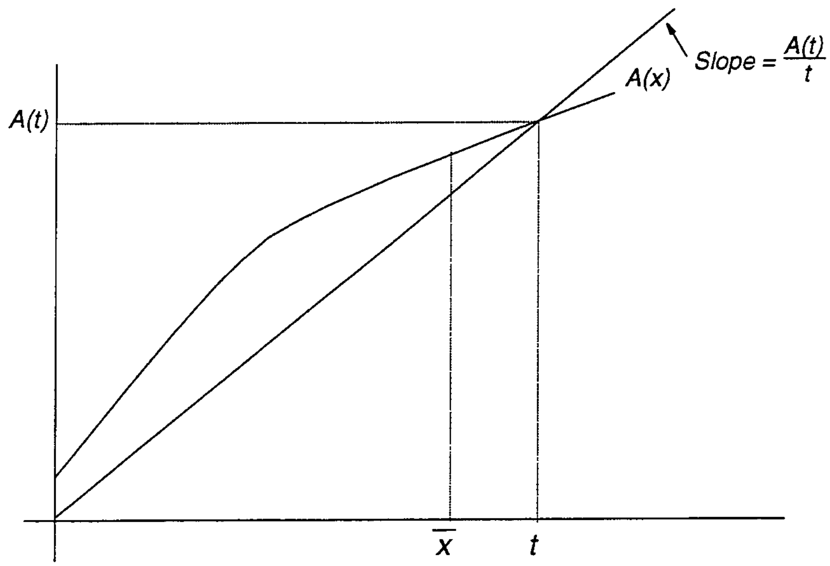

Figure 4.
Upper Bound for Concave Availability Function

Since $C(x^*) \leq C(x)$ for all x, we have

$$C(x^*) \leq C(x) = nP\left[x + \frac{1}{A(x)}\right] \leq nP\left[x + \frac{t}{A(t)x}\right] \textit{for } 0 \leq x \leq t.$$

Let $g(x) = x + \frac{t}{A(t)x}$. Then $g(x)$ is convex and has the minimum value at $\bar{x} = \sqrt{\frac{t}{A(t)}} < t$ and hence

$$C(x^*) \leq \min_{0 \leq x \leq t} nP\left[x + \frac{t}{A(t)x}\right] = 2nP\sqrt{\frac{t}{A(t)}}.$$

Since $\bar{x}A(\bar{x}) \geq (\bar{x})\frac{A(t)\bar{x}}{t} = \frac{A(t)}{t}(\bar{x})^2 = \frac{A(t)}{t}\frac{t}{A(t)} = 1$, we have $x^* \leq \bar{x}$. Therefore, an upper bound of x^* is $\bar{x} = \sqrt{\frac{t}{A(t)}}$ and an upper bound of total cost is $2nP\sqrt{\frac{t}{A(t)}}$.

3.2 S-Shaped Availability Function

In this section we try to find some properties of the optimal maintenance cost ratio x^* when the system availability function $A(x)$ is S-shaped as shown in Figure 1. If the weapon system has an S-shaped availability function, then $A(0)$ is very small. We will assume that $A(0) = 0$ for ease of computation.

Let t_o be a point such that $A(x) \leq \frac{A(t_o)}{t_o} \bullet x$ for all $x \geq 0$. That is, $A(x)$ is below the line $y = \frac{A(t_o)}{t_o} \bullet x$ for all $x \geq 0$ as is shown in Figure 5.

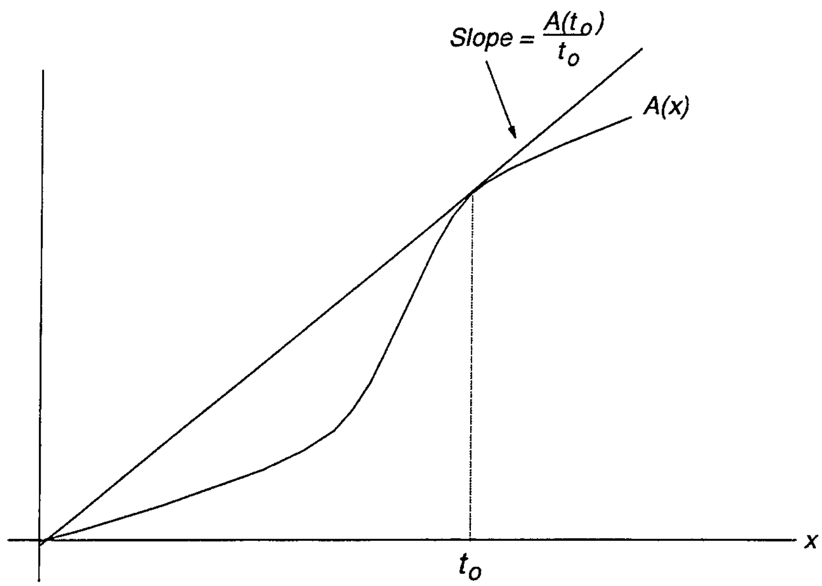

Figure 5.
S-Shaped Availability Function

3.2.1 Some Properties of the Optimal Maintenance Cost Ratio x^*

Case I: $0 \leq x \leq t_o$

For any x between 0 and t_o, we have

$$A'(x) \geq \frac{A(x)}{x} \ \ or \ \ \frac{A'(x)}{A^2(x)} \geq \frac{1}{xA(x)}.$$

If $x\,A(x) \leq 1$, then $\frac{A'(x)}{A^2(x)} \geq \frac{1}{xA(x)} \geq 1$ and hence $x^* \geq x$ from the convexity of $C(x)$ and optimality condition of x^*. Thus, if we know that $x\,A(x) \leq 1$ for some $x \leq t_o$, $x^*A(x^*) \geq 1$ or $x^* \geq t_o$. Figure 6 shows the availability function for Case I.

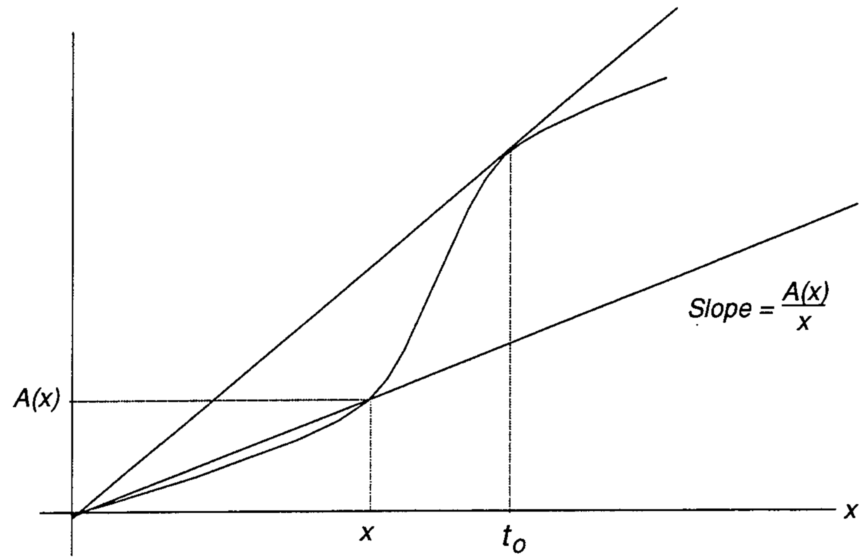

Figure 6.
S-Shaped Availability Function: Case I

Case II: $x \geq t_o$

For any $x \geq t_o$, we have

$$A'(x) \leq \frac{A(x)}{x} \text{ or } \frac{A'(x)}{A^2(x)} \leq \frac{1}{x\,A(x)}.$$

If $x\,A(x) \geq 1$, then $\frac{A'(x)}{A^2(x)} \leq 1$ and hence $x^* \leq x$. So, if we know that $x\,A(x) \geq 1$ for some $x \geq t_o$, then x is an upper bound of x^*. Figure 7 shows the availability function for Case II.

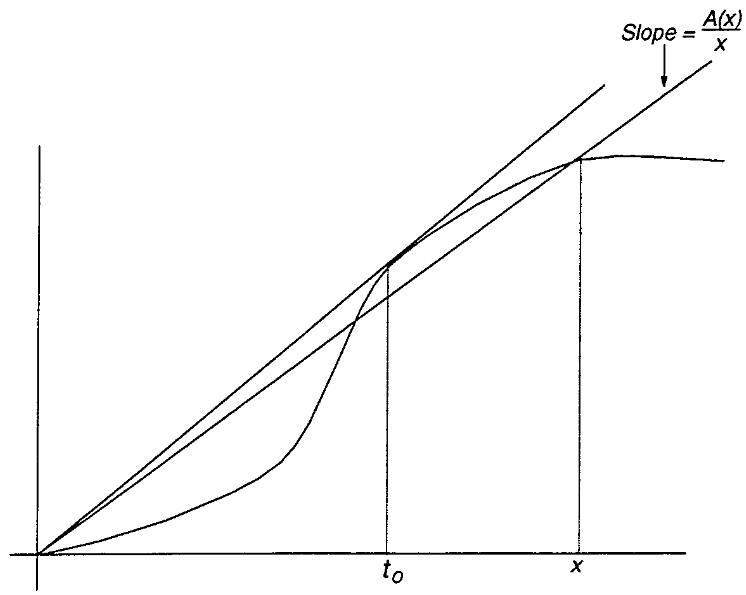

Figure 7.
S-Shaped Availability Function: Case II

3.2.2 An Upper Bound of the Optimal Cost Ratio

Suppose we know that $x^* \geq t_o$ and we have a point $(t, A(t))$ such that $t\,A(t) > 1$. Then we can show that the upper bound of the optimal maintenance cost ratio is $\bar{x} = \sqrt{\frac{t}{A(t)}} < t$.

Since we know that $x^* \geq t_o$, $t_oA(t_o) < 1$ because $x^* \leq t_o$ if $t_o\,A(t_o) \geq 1$. Let $\bar{x} = \sqrt{\frac{t}{A(t)}}$. Then $t_o < \bar{x} < t$, because $t\,A(t) > 1$ and $t_o\,A(t_o) < 1$. As is shown in Figure 8,

$$A(\bar{x}) \geq \frac{A(t)}{t}\,\bar{x}$$

and

$$\bar{x}\,A(\bar{x}) \geq \frac{A(t)}{t}(\bar{x})^2 = \frac{A(t)}{t}\frac{t}{A(t)} = 1.$$

Thus, since $\bar{x}A(\bar{x}) \geq 1$ *and* $\bar{x} > t_o$, we know that $x^* \leq \bar{x}$ by the property of optimal maintenance cost ratio.

Suppose we have field data $(t, A(t))$ with $t\,A(t) > 1$, but we do not know whether $x^* \geq t_o$ or $x^* \leq t_o$. Let $\bar{x} = \sqrt{\frac{t}{A(t)}}$.

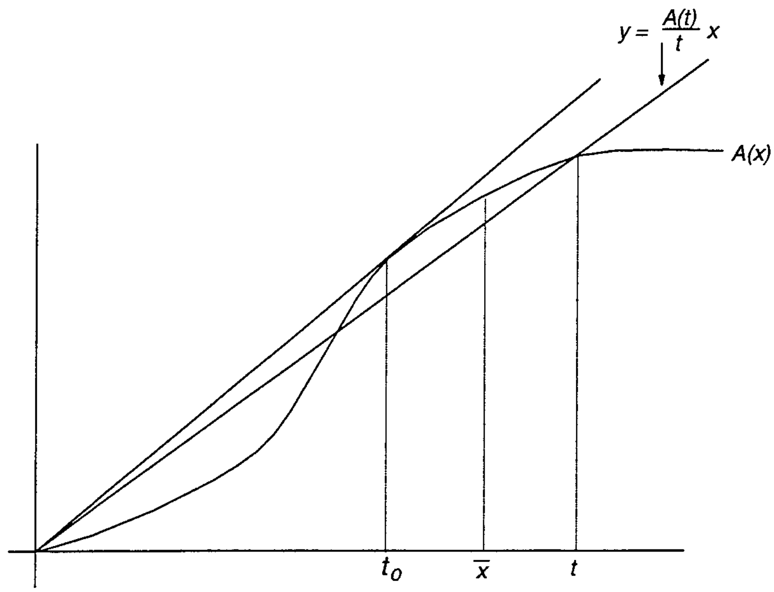

Figure 8.
Upper Bound for S-Shaped Availability Function

Case I: $A(\overline{x}) \geq \frac{A(t)}{t}\,\overline{x}$.

If $A(\overline{x}) \geq \frac{A(t)}{t}\,\overline{x}$, then $t > t_o$ and $C(x^*) \leq C(\overline{x}) = nP\left[\overline{x} + \frac{1}{A(\overline{x})}\right] \leq 2nP\sqrt{\frac{t}{A(t)}}$.

If $x^* \geq t_o$, then we know that $x^* \leq \overline{x} = \sqrt{\frac{t}{A(t)}}$. Now suppose that $x^* < t_o$.

Since $C(x^*) = nP\left[x^* + \frac{1}{A(x^*)}\right] \leq 2nP\sqrt{\frac{t}{A(t)}}$, we have

$$x^* \leq 2\sqrt{\frac{t}{A(t)}} - \frac{1}{A(x^*)} \leq 2\sqrt{\frac{t}{A(t)}} - \frac{1}{A(t)}.$$

Because we do not know whether $x^* \geq t_o$ or $x^* \leq t_o$, we only have

$$x^* \leq \max\left\{\sqrt{\frac{t}{A(t)}},\, 2\sqrt{\frac{t}{A(t)}} - \frac{1}{A(t)}\right\} = 2\sqrt{\frac{t}{A(t)}} - \frac{1}{A(t)}.$$

Thus an upper bound of x^* is $\overline{x} = 2\sqrt{\frac{t}{A(t)}} - \frac{1}{A(t)}$ and an upper bound of total cost $C(x)$ is $2nP\sqrt{\frac{t}{A(t)}}$.

Case II: $A(\overline{x}) < \frac{A(t)}{t}\,\overline{x}$.

In this case, we cannot find an upper bound of x^* by using only one data point $(t, A(t))$. But we know that $x^*A(x^*) \geq 1$ and hence $x^* \geq \frac{1}{A(x^*)} > 1$. So, if the value of t $A(t)$ is close to one for sufficiently large A(t), it is reasonable to assume that $A(\bar{x}) \geq \frac{A(t)}{t}\,\bar{x}$ and hence an upper bound of x^* is $\bar{x} = 2\sqrt{\frac{t}{A(t)}} - \frac{1}{A(t)} < t$.

3.3 Summary

If we know that $x^* \geq t_o$ and $t\,A(t) > 1$ for some t_o, then an upper bound of x^* is $\bar{x} = \sqrt{\frac{t}{A(t)}}$.

Let t be a number such that $t\,A(t) > 1$. Then, either $x^* \leq \bar{x} = \sqrt{\frac{t}{A(t)}}$ or $x^*A(x^*) \geq 1$. Moreover, if $A(\bar{x}) \geq \frac{A(t)}{t}\bar{x}$, then an upper bound of x^* is $\bar{x} = 2\sqrt{\frac{t}{A(t)}} - \frac{1}{A(t)}$.

Table 1 shows the upper bounds of x^* and $C(x^*)$ for the weapon systems according to their level of technology for given data $(t, A(t))$ with $t\,A(t) > 1$. For a simple low-technology weapon system whose components are relatively reliable and inexpensive to maintain, the initial availability $A(o)$ is determined by the mean time between failures (MTBF) and is typically higher than for a more complex high-technology system. In this case, the marginal effect of additional maintenance resources is decreasing, which means that $A(x)$ is concave and an upper bound of x^* is $\sqrt{\frac{t}{A(t)}}$. For the high-tech weapon systems, the availability function may be S-shaped and may not satisfy the condition that $A(\bar{x}) \geq \frac{A(t)}{t}\bar{x}$. So we cannot identify an upper bound of x^* at this point. However, if $t\,A(t)$ is close to one, an upper bound of x* is $\bar{x} = 2\sqrt{\frac{t}{A(t)}} - \frac{1}{A(t)} < t$.

Table 1.
Summary of Results for Upper Bound of Maintenance Cost Ratio

Level of technology	Upper bound of x^*	Upper bound of $C(x^*)$
Low	$\sqrt{\frac{t}{A(t)}}$	$2\sqrt{\frac{t}{A(t)}}\,nP$
Medium	$2\sqrt{\frac{t}{A(t)}} - \frac{1}{A(t)}$	$2\sqrt{\frac{t}{A(t)}}\,nP$
High	?	?

4.0 CANNIBALIZATION

4.1 Optimal Maintenance Cost Model Using Cannibalization

Suppose an end item has failed and cannot be fixed in a short time because of lack of spares or other resources. But suppose we can use cannibalization to fix this end item.

By introducing cannibalization as a means of maintenance, we have three sources of maintenance for failed end items in the field:

- *Standard maintenance.* Fix the failed end items by using the repair parts in stock. The capacity of standard maintenance is determined by the annual maintenance budget $\alpha = x/T$.

- *Cannibalization.* Fix the failed end items by cannibalization if we cannot fix them in a short time by standard maintenance.

- *Maintenance float.* Replace the failed end items by M/F if they cannot be fixed either by standard maintenance or by cannibalization. But the failed end items replaced by M/F are repaired only by using the standard maintenance procedure.

We will assume $A(x)$ is made concave by the use of cannibalization.

Suppose m end items fail and we assume that km end items can be fixed by cannibalization, where k is the reproduction rate of the failed end items by using cannibalization. That is, we can fix km end items by using m failed end items. Let $M(x)$ be the expected number of available end items at any time. Then

$$M(x) = N(x)A(x) + n[1 - A(x)]k. \qquad \text{[Eq. 6]}$$

The first term in the right-hand side of Equation 6 is the number of available end items without using cannibalization, and the second term is the number of end items that can be reproduced (or fixed) by using cannibalization. Since the target number of available end items is n, we have the following relation:

$$N(x)A(x) + n[1 - A(x)]k = n \qquad \text{[Eq. 7]}$$

or

$$\begin{aligned} N(x) &= \frac{n[1-k+kA(x)]}{A(x)} \\ &= \frac{n}{A_o(x)}, \end{aligned}$$

where

$$A_o(x) = \frac{A(x)}{1-k+kA(x)}. \qquad \text{[Eq. 8]}$$

Then the minimization of $C(x)$ with cannibalization is equivalent (same cost and same number of available end items) to the minimization of $C(x)$ with availability function $A_o(x)$ without cannibalization (see Figure 9).

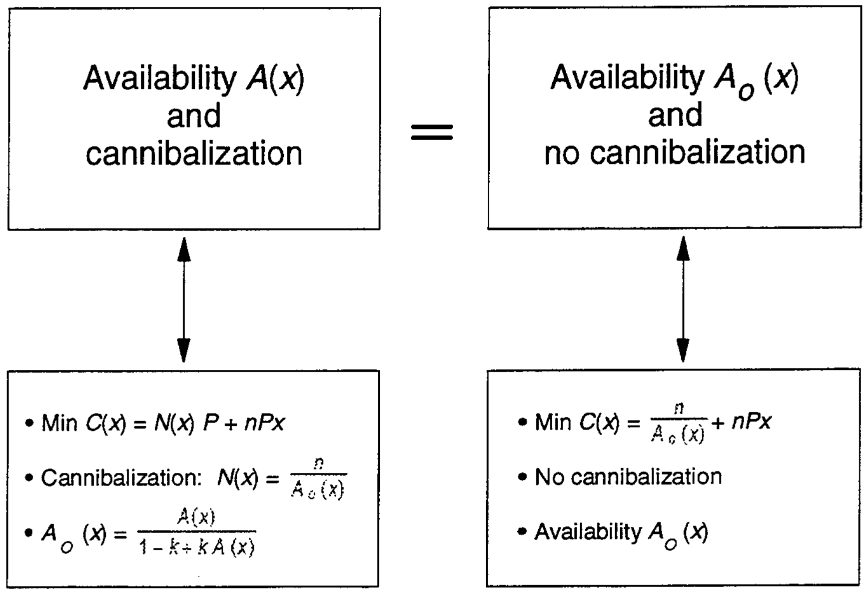

Figure 9.
Equivalence of Availability Functions

4.2 Reproduction Rate of Cannibalization (k-Factor)

Suppose an end item fails and cannot be fixed by standard maintenance. Then the failure of the end item is due to component i with probability $\frac{F_i}{\sum_j F_j}$, where F_j is the probability that component j is backordered. (We assume that components fail independently.) If m end items fail, then the expected number of component i that can be used for cannibalization is

$$m\left[1 - \frac{F_i}{\sum_j F_j}\right].$$

The expected number of end items that can be fixed by using m end items is, at least,

$$\min_i \left\{ m\left[1 - \frac{F_i}{\sum_j F_j}\right]\right\}$$

and hence

$$k \geq \min_i \left\{ 1 - \frac{F_i}{\sum_j F_j} \right\} = 1 - \frac{\max_i F_i}{\sum_j F_j}.$$

If all the components have similar not-mission-capable supply (NMCS) rates, then $k = 1 - \frac{1}{K}$, where K is the number of components in the system.

The k-factor is highly dependent on $\max_i F_i$. But k is sufficiently large even if $\max_i F_i$ is large. If only one or two components have high probability of backorders and all the other components have relatively low probability rates, we can reduce $\max_i F_i$, so that k is large enough, by investing relatively small amounts of maintenance resources in these high-probability-rate components. However, this is a very rare case in real situations. In general, several components have similar probabilities. Suppose, for example, we have the probability rates shown in Table 2 for the components. In this case $k \geq 0.85$.

Table 2.
Example of Component Backorder Rates

Component	Backorder rate per year	Mean time between backorders (months)
1	4	3
2	4	3
3	3	4
4	3	4
5	3	4
6	2	6
7	2	6
8	2	6
9	2	6
10	2	6
Others	Almost 0	–

Moreover, the components or repair parts with high probability rates (and also high failure rates) are mostly cheap ones compared to the low-failure-rate items. So, we can increase the availabilities of these components with relatively small amounts of resources to make k large enough.

4.3 Property of $A_o(x)$ with Fixed k

By using cannibalization, we have availability $A_o(x)$, given by

$$A_o(x) = \frac{A(x)}{1-k+kA(x)}.$$

It is easy to show that $A_o(x) \geq A(x)$ for all x and $A_o(x)$ is an increasing function of x.

The second derivative of $A_o(x)$ is given by

$$A_o''(x) = \frac{1-k}{[1-k+kA(x)]^3}\left\{A''(x)[1-k+kA(x)] - 2k[A'(x)]^2\right\}.$$

Since $\ln A(x)$ is concave, $[A'(x)]^2 \geq A''(x)A(x)$ for all $x \geq 0$ and hence we have

$$A''(x) \leq \frac{(1-k)A''(x)}{[1-k+kA(x)]^3}[1-k-kA(x)].$$

So $A_o''(x)$ has the following properties:

If $A''(x) \leq 0$, then $A_o''(x) \leq 0$.
If $A''(x) \geq 0$ and $k \geq \frac{1}{1+A(x)}$, then $A_o''(x) \leq 0$.
If $A''(x) \geq 0$ and $k < \frac{1}{1+A(x)}$, then $A_o''(x) \leq \frac{A''(x)}{1-k+kA(x)}$.

If $A(x)$ is small, then $A(x)$ is approximately linear, especially for the high-tech weapon systems, and hence $A''(x) \approx 0$ and $A_o''(x) \leq 0$. In the midranges of $A(x)$, k is large enough so that $k \geq \frac{1}{1+A(x)}$. For high values of $A(x)$, $A(x)$ is concave and hence $A_o(x)$ is concave. Therefore, we can assume that $A_o(x)$ is concave, at least approximately.

Figure 10 shows that $A_o(x)$ is nearly concave in the typical situations.

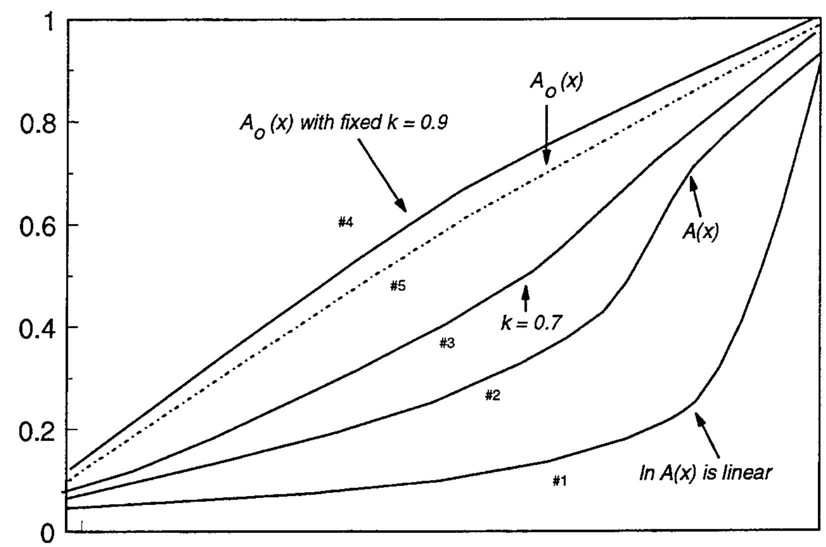

Figure 10.
Example of Typical Availability Function

Curve 1 in Figure 10 is the extreme case of system availability that $lnA(x)$ is linear. So, the typical availability curve lies above curve 2. Curves 3 and 4 denote the availability curves when we use cannibalization with $k = 0.7$ and $k = 0.9$, respectively. Then $A_o(x)$, shown by curve 5, lies between curves 3 and 4 and is nearly concave.

4.4 An Upper Bound of Optimal Maintenance Cost Ratio Using Cannibalization

Suppose $A_o(x)$ is concave and we have a field data $t\,A(t) > 1$ with $t\,A(t) > 1$. Then an upper bound of optimal maintenance cost ratio is

$$\bar{x} = \sqrt{\frac{t}{A(t)}}$$

and an upper bound of total cost is

$$\bar{C} = 2\sqrt{\frac{t}{A(t)}}\,nP.$$

If $t = 2$ and $A(t) = 0.9$, for example, then $\bar{x} = 1.491$ and $\bar{C} = 2.981\,nP$. The reduction in total costs is 4.2 percent and the reduction in maintenance costs (including M/F costs) is 6.2 percent.

If we know the value of k, the upper bound of optimal maintenance cost ratio is

$$\bar{x} = \sqrt{\frac{t}{A_o(t)}} = \sqrt{\frac{t}{A(t)}[1 - k + kA(t)]}$$

and an upper bound of total cost is

$$\bar{C} = 2\sqrt{\frac{t}{A_o(t)}}\ nP = 2\sqrt{\frac{t}{A(t)}[1 - k + kA(t)]}\ nP.$$

If $t = 2$, $A(t) = 0.9$, and $k = 0.7$, then $\bar{x} = 1.438$ and $\bar{C} = 2.875\,\text{np}$. The reduction in total costs is 7.6 percent and the reduction in lifetime maintenance costs (including M/F costs) is 11.2 percent.

4.5 Examples

For ground weapon systems such as the M1 Abrams tank and the M113 A3 armored personnel carrier (APC), current maintenance cost ratios are slightly above 2.0 and average availabilities are 0.90 to 0.95. For the Multiple Launch Rocket System (MLRS), the current maintenance cost ratio is also slightly greater than 2.0. For the rotary-wing aircraft, the maintenance cost ratio is about 1.5 to 1.6. Maintenance costs include repair parts costs (consumption plus average stock), depot maintenance costs, and intermediate maintenance costs. Maintenance personnel costs at the unit level are not included. Table 3 shows the maintenance cost ratios of several weapon systems.

Table 3.
Weapon System Maintenance Costs

Weapon system	Weapon system unit cost[a] ($)	Average lifetime costs per system[b] ($)	Maintenance cost ratio
M1 Abrams tank	2,300,000	4,674,089	2.03
M113 A3 APC	180,000	362,248	2.01
MLRS	3,000,000	6,044,900	2.01
AH-64 helicopter	10,680,000	16,000,000	1.50
UH-60 helicopter	5,800,000	8,772,000	1.51

[a]Weapon system unit cost is the average procurement cost of the end item.

[b]Average lifetime maintenance costs per system are total lifetime (20 years) maintenance costs of the weapon systems divided by the target number of available weapon systems.

For the M1, M113 A3, and MLRS, the upper bounds of the optimal maintenance cost ratio is as shown in Table 1 of Section 3.0. For the AH-64 and UH-60, an upper bound of the optimal maintenance cost ratio (when $k = 0.8$) is $\bar{x} = 1.255$, and an upper bound of total cost is $\bar{C} = 2.510\,nP$. The reduction in maintenance costs (including M/F costs) is 13.7 percent.

4.6 Summary

By using cannibalization, we can reduce the total life-cycle costs (or maintenance cost including M/F costs) of the weapon systems. Let

$$A_o(x) = \frac{A(x)}{1-k+kA(x)}.$$

Then, by using cannibalization, we can have an equivalent optimization problem with availability function $A_o(x)$.

We have shown that $A_o(x)$ is nearly concave for all $x \geq 0$ and, hence, we can use the upper bound of optimal maintenance cost ratio $\bar{x}$ where $\bar{x} = \sqrt{\frac{t}{A(t)}}$. Or, we can make $A_o(\bar{x}) \geq A(t)\frac{\bar{x}}{t}$, which might not be satisfied for the original availability function $A(x)$. Therefore, if we use cannibalization, the upper bound of optimal maintenance cost ratio is $\bar{x} = \sqrt{\frac{t}{A(t)}}$ for given data $(t, A(t))$ with $t\,A(t) > 1$. Moreover, if we know the value of k, then we can improve the upper bound to $\bar{x} = \sqrt{\frac{t[1-k+kA(t)]}{A(t)}}$.

5.0 Conclusions

Suppose field data show that the average system availability is $A(t)$ with lifetime maintenance cost ratio t, with $t\,A(t) \geq 1$. If $A(x)$ is a concave function of x, then the upper bound of the optimal maintenance cost ratio is $\bar{x} = \sqrt{\frac{t}{A(t)}}$.

In many cases, especially for high-tech weapon systems, the availability function $A(x)$ is S-shaped. If $A(\bar{x})$is large enough to satisfy $A(\bar{x}) \geq A(t)\frac{\bar{x}}{t}$, then the upper bound of x^* is $2\sqrt{\frac{t}{A(t)}} - \frac{1}{A(t)}$. If $A(\bar{x})$ is small, then cannibalization should be used to increase weapon system availability.

By using cannibalization, we have a new availability function $A_o(x)$, defined in the section on cannibalization, and an equivalent optimization problem with cannibalization. In this case, the upper bound of x^* is $\bar{x} = \sqrt{\frac{t}{A(t)}}$, which is the same as that of the concave availability function $A(x)$. If we know the reproduction rate of cannibalization, then we can further improve the upper bound of the maintenance cost ratio.

These results demonstrate the value of cannibalization, especially for high-tech weapon systems, for reducing total life-cycle costs while maintaining the target number of available end items in the field.

If $t\,A(t) > 1$, then the lifetime maintenance costs (nPt) are greater than the weapon system procurement costs ($nP/A(t)$). The results of this study show that it is economical to reduce lifetime maintenance costs and increase the number of end items until the two costs (maintenance costs and system procurement costs) become the same when the availability function $A(x)$ is concave. This means that the lifetime maintenance costs should not be higher than the weapon system procurement costs (or a cheaper solution would be to buy more units of the weapon system). This might be true for the S-shaped availability cases unless the availability of the end items decreases drastically for small cuts in the maintenance resources.

This study shows that x^* is more likely to be less than t if $t\,A(t) > 1$ even for the S-shaped availability functions. In this case, we should examine the effectiveness of the current maintenance policies and resource allocations. The use of cannibalization ensures that we can reduce current maintenance costs if $t\,A(t) > 1$.

If lifetime maintenance costs (excluding fixed costs, such as test equipment and maintenance facilities) are greater than the weapon system procurement costs, we should improve maintenance management processes. We should also perform tradeoff analysis among the investments in maintenance resources and in end-item procurements to increase system availability with less cost. Finally, we should use cannibalization, if necessary, to increase system availability.

Glossary

APC	=	armored personnel carrier
M/F	=	Maintenance Float
MLRS	=	Multiple Launch Rocket System
MTBF	=	mean time between failures
NMCS	=	not-mission-capable supply
T/E	=	Table of Equipment

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This paper presents a theoretical discussion of the relationship between weapon system life cycle maintenance costs and procurement costs. For certain situations, it derives upper bounds for the optimal allocation of maintenance and procurement resources, while maintaining a targeted number of operationally available weapon systems.

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